Coherence without Rationality at the Zero Lower Bound

Guido Ascari[1] Sophocles Mavroeidis[2] Nigel McClung[3]

[1]University of Pavia and DNB [2]University of Oxford

[3]Bank of Finland

Bank of Finland and CEPR Joint Conference
September 16-17, 2022

*The views expressed in this presentation are the authors' views and not the views of the Bank of Finland, DNB, or the Eurosystem.

Existence and the ZLB

Motivation:

- Many advanced economies stuck at ZLB in recent decades.
- Issues with existence/uniqueness of rational expectations equilibrium (e.g. Ascari and Mavroeidis, 2022).

A ZLB Puzzle:

- A large/persistent demand shock $\implies$ binding ZLB.
- Rational agents expect long period of high real rates $\implies$ strong income effects (Bilbiie, forth.).
- Strong income effect $\implies$ high inflation and positive nominal rate. No equilibrium!

Existence and the ZLB

Question: Are rational agents too sophisticated/forward-looking for their own good? What are equilibrium properties away from FIRE?

This paper:

- Derives existence/uniqueness results away from FIRE in a stochastic model with occasionally-binding ZLB constraint.
- Deviations from RE that dampen expectations mitigate existence/uniqueness concerns.
  1. Discounting (e.g. Gabaix, 2020; Woodford and Xie, 2020; Angeletos and Lian, 2018).
  2. Adaptive Learning + Misspecified Forecasts

Environment

Consider a New Keynesian model:

$$\begin{aligned} x_t &= E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1}) + \epsilon_t \\ \pi_t &= \lambda x_t + \beta E_t \pi_{t+1} \\ i_t &= \max\{\psi \pi_t, -\mu\} \end{aligned}$$

- $\epsilon_t \in \{\epsilon^H, \epsilon^L\} \sim MC(q, p)$
- $\epsilon^L \leq 0 \leq \epsilon^H$
- p (q) is the persistence of the low (high) state

A Simple Example

Suppose $q = 1$ and $\epsilon^H = 0$. The model can be simplified:

$$\begin{aligned} x_t &= \nu(p) E_t x_{t+1} - \sigma \max\{\frac{\psi\lambda}{1-\beta p} x_t, -\mu\} + \epsilon_t \\ \nu(p) &= 1 + \frac{\psi\sigma}{1-\beta p} > 1 \end{aligned}$$

A large negative shock ($|\epsilon^L|$ large) $\Longrightarrow$ binding ZLB and the (MSV) solution:

$$x_t = \frac{1}{1 - p\nu(p)}(\sigma\mu + \epsilon^L)$$

Non-existence

The solution:

$$x_t = \frac{1}{1 - p\nu(p)}(\sigma\mu + \epsilon^L)$$

- The expectations term $E_t x_{t+1}$ can explode if $p\nu(p) > 1$ (income effect).
- Need to restrict p or ϵ^L for the solution to exist.

Non-existence

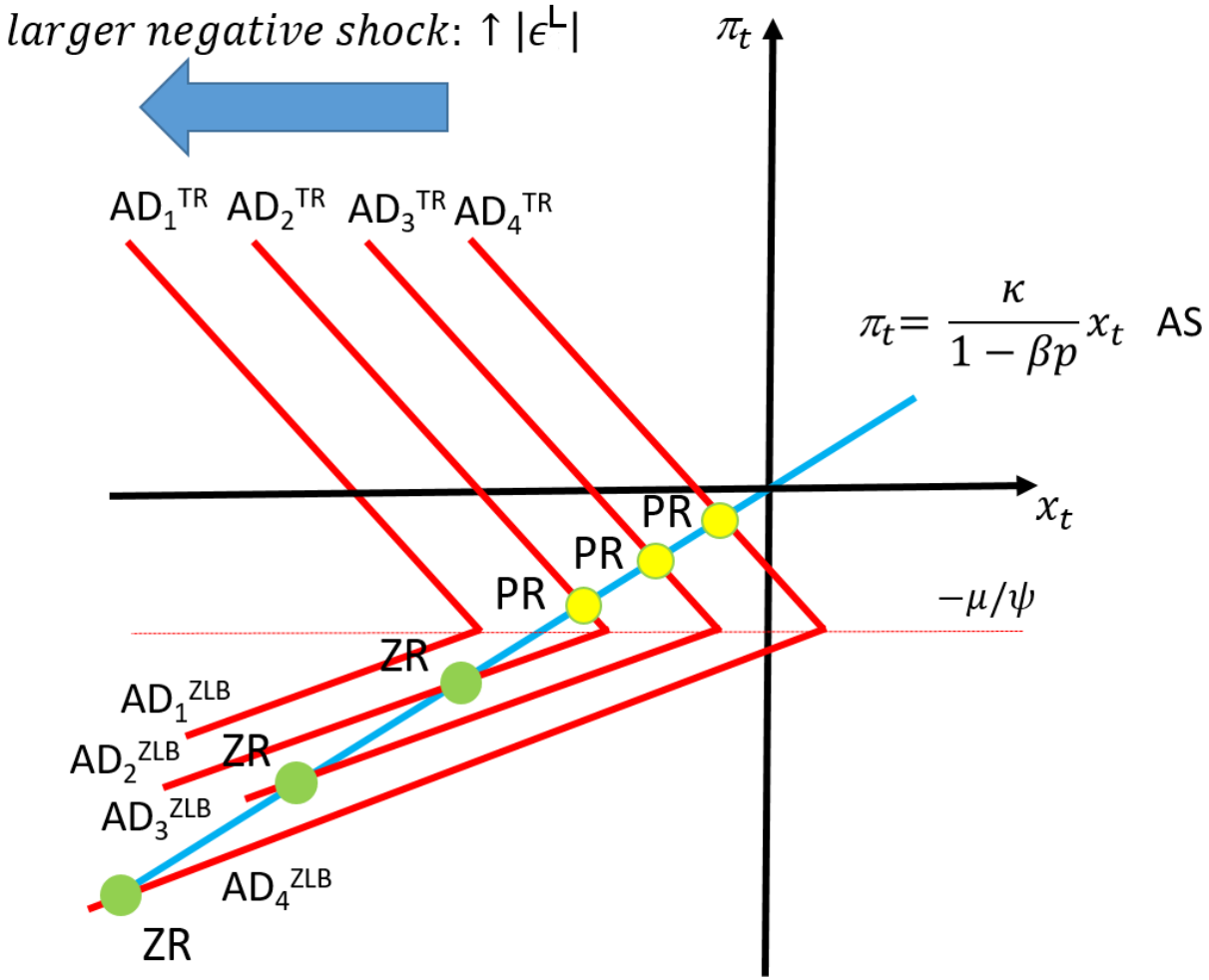


Figure: $p\nu(p) > 1$

Non-existence

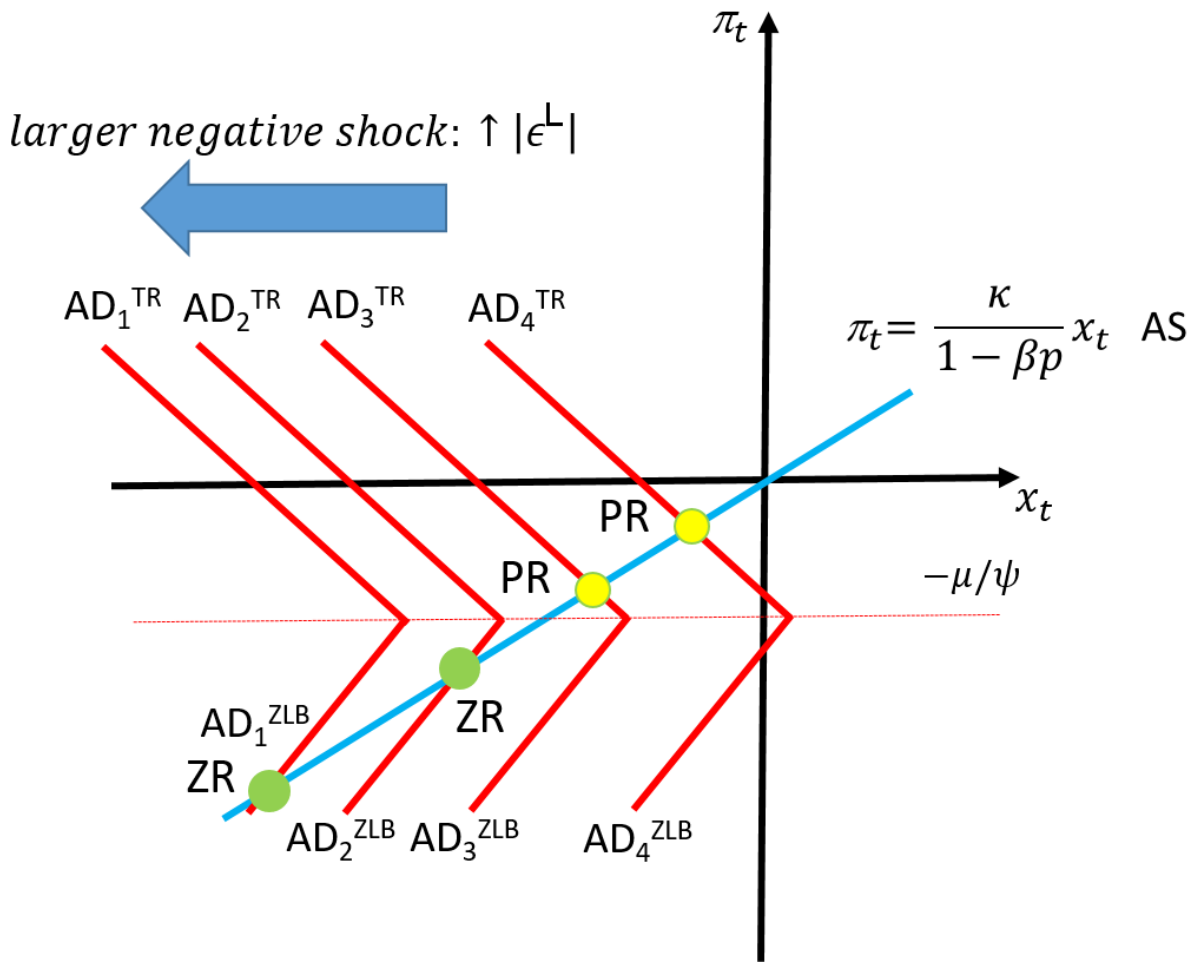


Figure: $p\nu(p) < 1$

Non-existence

Some general points about existence:

1. **Proposition:** MSV solution exists iff $\epsilon^L > \bar{\epsilon}_{REE}(p, q, \epsilon^H)$.

Non-existence

Some general points about existence:

1. **Proposition:** MSV solution exists iff $\epsilon^L > \bar{\epsilon}_{REE}(p, q, \epsilon^H)$.

2. Need to restrict demand shock to get **sunspot solutions**
   - E.g. Mertens and Ravn (2014), Nakata and Schmidt (forth.), Bilbiie (forth.) generate persistent liquidity traps using discrete-valued sunspot shocks.

Non-existence

Some general points about existence:

1. **Proposition:** MSV solution exists iff $\epsilon^L > \bar{\epsilon}_{REE}(p, q, \epsilon^H)$.

2. Need to restrict demand shock to get **sunspot solutions**
   - E.g. Mertens and Ravn (2014), Nakata and Schmidt (forth.), Bilbiie (forth.) generate persistent liquidity traps using discrete-valued sunspot shocks.

3. Numerical evidence: other non-fundamental equilibria exist only if $\epsilon^L > \bar{\epsilon}_{REE}(p, q, \epsilon^H)$.
   - Analytical results for the MSV/sunspot cases described above.

Dampening Income/GE Effects

Under FIRE, strong expectations feedback (income effect) strains existence. Consider an ad hoc dampening of expectations:

$$\begin{aligned} x_t &= \nu(p)\hat{E}_t x_{t+1} - \sigma \max\{\frac{\psi\lambda}{1-\beta p}x_t, -\mu\} + \epsilon_t \\ \hat{E}_t x_{t+1} &= mE_t x_{t+1}, m \in [0,1) \end{aligned}$$

For large shock:

$$x_t = \frac{1}{1 - mp\nu(p)}(\sigma\mu + \epsilon^L)$$

Result: small m ensures solution for any p or ϵ^L.

Dampening Expectations:

1. Discounted Expectations
2. Adaptive Learning and Misspecified Forecasts

Discounting of Expectations

Consider the bounded rationality model:

$$\begin{aligned} x_t &= M E_t x_{t+1} - \sigma(i_t - N E_t \pi_{t+1}) + \epsilon_t \\ \pi_t &= \lambda x_t + M_f \beta E_t \pi_{t+1} \\ i_t &= \max\{\psi \pi_t, -\mu\} \end{aligned}$$

- E.g. Gabaix (2020), Woodford & Xie (2020), Angeletos & Lian (2018).
- **Proposition:** a bounded rationality equilibrium exists for any $p, q, \epsilon^L, \epsilon^H$ if

$$(M-1)(1-M_f\beta) + \lambda\sigma N < 0$$

Imperfect Knowledge and Model Misspecification

Suppose agents learn to forecast adaptively:

$$\hat{E}_t z_{t+1} \quad = \quad g_t z_{t-1} + (1 - g_t)\hat{E}_{t-1} z_t$$

where $z = \pi, x$.

- Forecasting model is misspecified (it omits state variable ϵ_t).
- Set $E_t z_{t+1} = \hat{E}_t z_{t+1}$ in NK model and compute temporary equilibrium.

Question: Will beliefs about *average* inflation/output converge to a self-confirming **restricted perceptions equilibrium (RPE)** (i.e. $\hat{E}_t z_{t+1} \to E(z) = \bar{z}$)?

Restricted Perceptions Equilibrium

Proposition: For given p, q and $\epsilon^H \geq 0$:

i. RPE exists if and only if $\epsilon^L > \bar{\epsilon}_{RPE}(p, q, \epsilon^H)$.

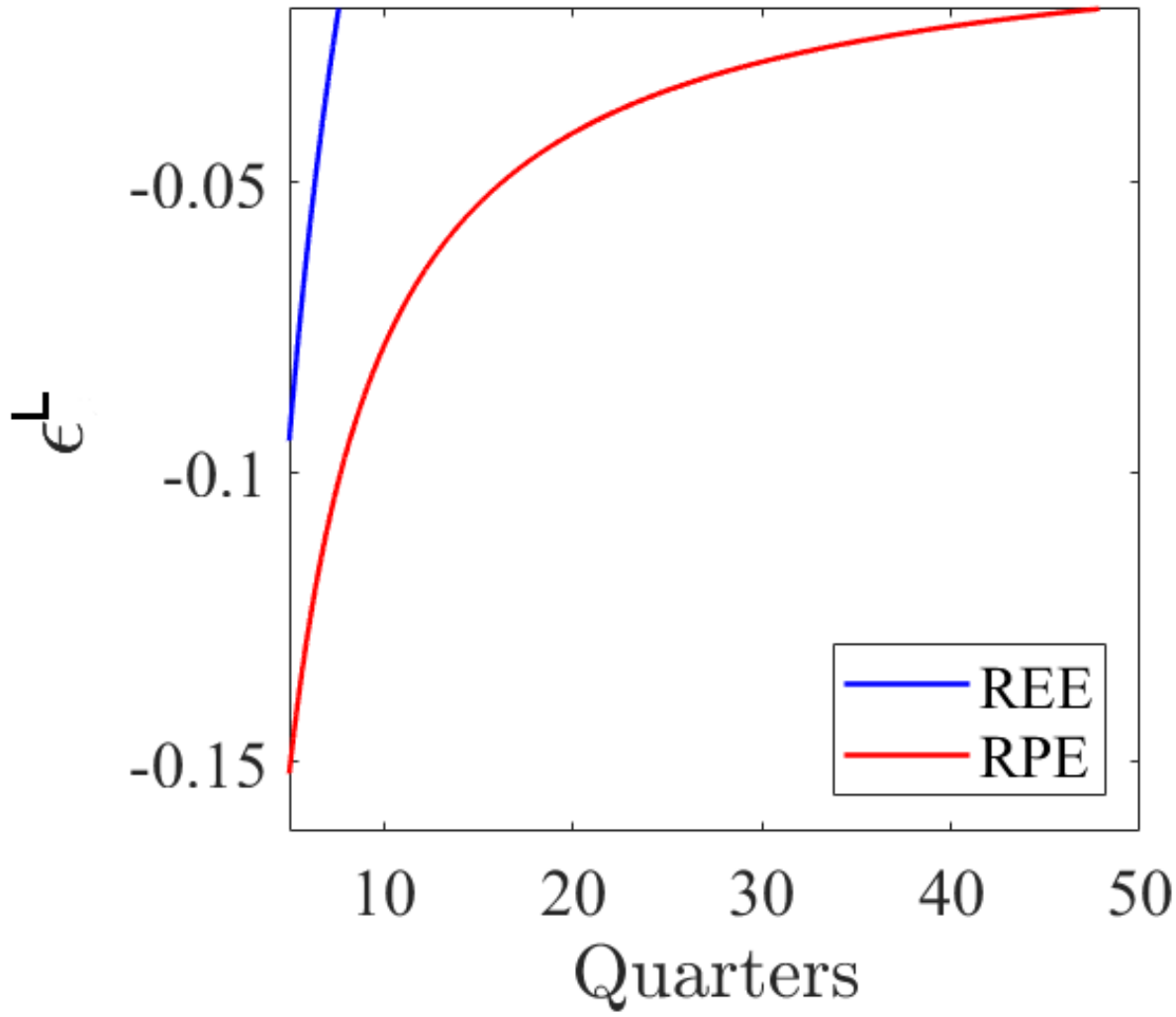
ii. $\bar{\epsilon}_{REE}(p, q, \epsilon^H) > \bar{\epsilon}_{RPE}(p, q, \epsilon^H)$ if and only if $p + q > 1$.

Intuition: weaker feedback from expectations to equilibrium outcome in RPE $\iff$ $p + q > 1$.

- REE forecast in low state: $E_t z_{t+1} = pz_L + (1-p)z_H$.
- RPE forecast in low state: $\hat{E}_t z_{t+1} = \bar{z} = \bar{p}z_L + (1-\bar{p})z_H$.
- Less feedback from expectation in RPE $\iff$ $\bar{p} = \frac{1-q}{2-p-q} < p$ $\iff$ $p + q > 1$.

Liquidity Trap Duration

RPE can feature *highly* persistent liquidity trap episodes



Equilibrium Selection and Uniqueness

Deviations from RE select equilibrium.

1. Either no or multiple MSV solutions exist (Ascari and Mavroeidis, 2022), we find there is a unique E-stable MSV.
   - Extends Christiano et al. (2018) to recurring ZLB ($q < 1$).
   - Sunspot equilibria (e.g. Mertens and Ravn, 2014) are generally not E-stable $\Longrightarrow$ deflationary spirals if ZLB is *recurring.*

2. We prove there is a unique E-stable RPE.

3. Sufficient discounting (e.g. Gabaix, 2020) ensures a unique solution.

Equilibrium Selection and Uniqueness

Learning view: RPE is a plausible explanation for *highly* persistent liquidity traps.

- Sunspot equilibria may not be.

Bounded rationality (e.g. Gabaix) view: sufficient discounting ensures a unique solution.

Coherence without Rationality

- Generating persistent liquidity traps in a FIRE framework with shocks is challenging.
  - Strong *rational* expectations feedback contributes to this problem.
- Dampened expectations can lead to **coherence without rationality**: existence of a non-rational equilibrium under conditions that preclude standard rational equilibria.
- Departures from RE generate highly persistent liquidity trap events.

Conclusions

New Keynesian model with ZLB constraint:

- Known issues with *rational* equilibrium existence/uniqueness.
- Deviations from RE resolve many of these issues.

Extensions:

- Policy Transmission in RPE.
  - Forward guidance and learning (Eusepi, Gibbs & Preston, 2022).
- Deviations that *amplify* expectations make existence harder.
- Characterizing the full rational solution space.
  - More sophisticated approach to learning (e.g. Ashwin, Beaudry, Ellison, 2021).

Lagged Expectation Equilibrium

- Consider Eggertsson-Woodford set-up, and suppose agents observe ϵ^{t-1} but *not* ϵ_t at time-t
  - Rational forecast: $E_t z_{t+1} = p z_1 + (1-p) z_2$
  - Lagged info forecast: $\hat{E}_t z_{t+1} = p^2 z_1 + (1-p) z_2$

´

- At ZLB, temporary ZLB output in **lagged expectations equilibrium (LEE)** is

$$x_t = \frac{1}{1 - p^2 \nu(p^2)} (\sigma \mu + \epsilon^1)$$

- Since $p^2 \nu(p^2) < p \nu(p)$, a LEE may exist when no MSV exists.

Is the RPE Reasonable?

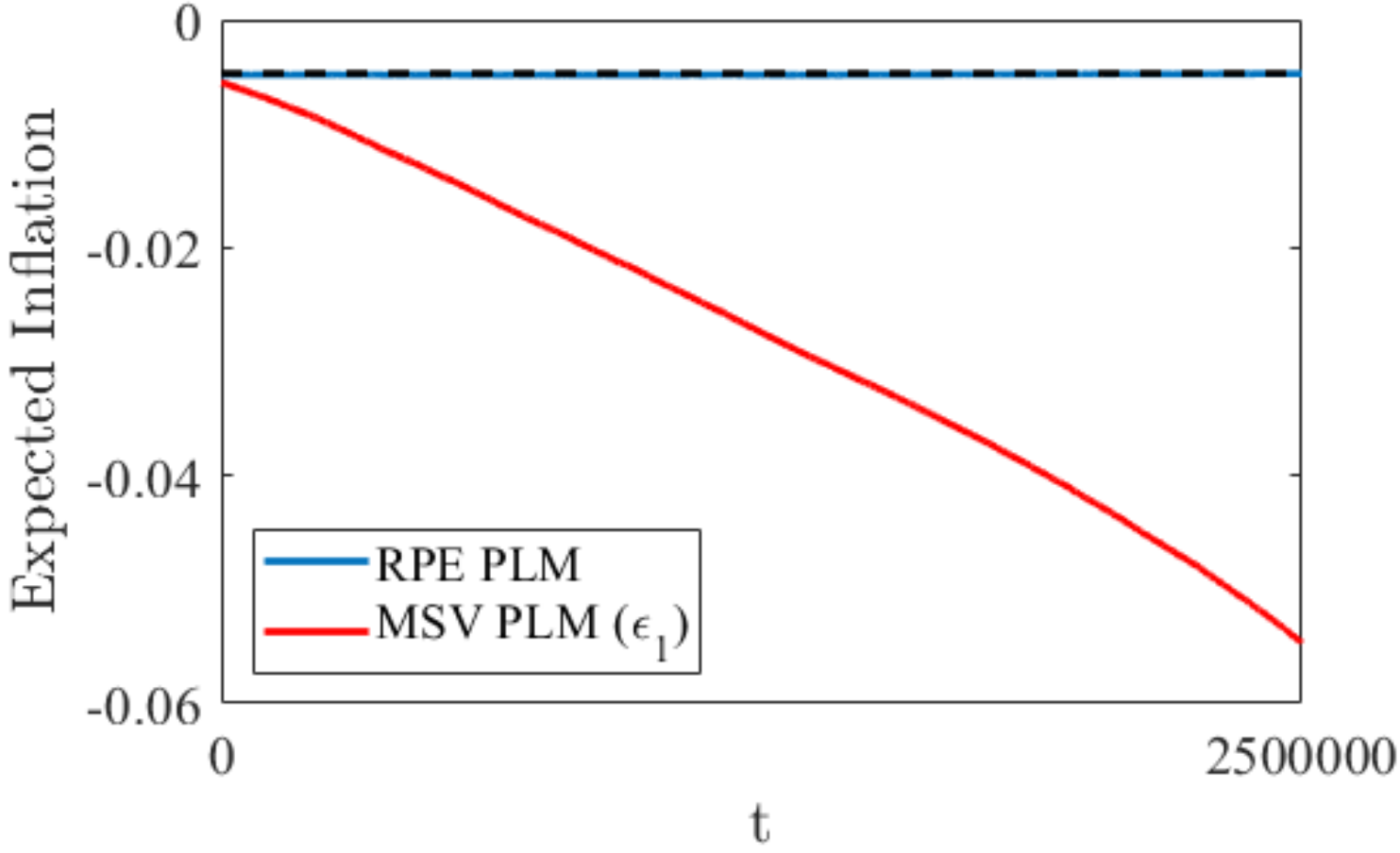
1. RPE forecasts are badly misspecified–but what's the better alternative given rational incoherence?
   - The following forecasting models cannot yield **self-confirming beliefs** about the 2-state dynamics or serial correlation in incoherent models (if agents observe current ϵ_t, Y_t.):

$$
\begin{aligned}
Y_t^e &= a_{\epsilon^{t-k}} \\
Y_t^e &= a_{\epsilon^{t-k}} + b\epsilon^{t-k} \\
Y_t^e &= a + b_{\epsilon^{t-k}}\epsilon^{t-k} \\
Y_t^e &= a_{\epsilon^{t-k}} + b_{\epsilon^{t-k}}\epsilon^{t-k} \\
Y_t^e &= a + b\epsilon^{t-k} \\
Y_t^e &= a + bY_{t-1} \\
Y_t^e &= a_{s_{t-k}} + b_{s_{t-k}}Y_{t-1}
\end{aligned}
$$

   where $k = 0, 1$ and s_t is the endogenous policy regime.

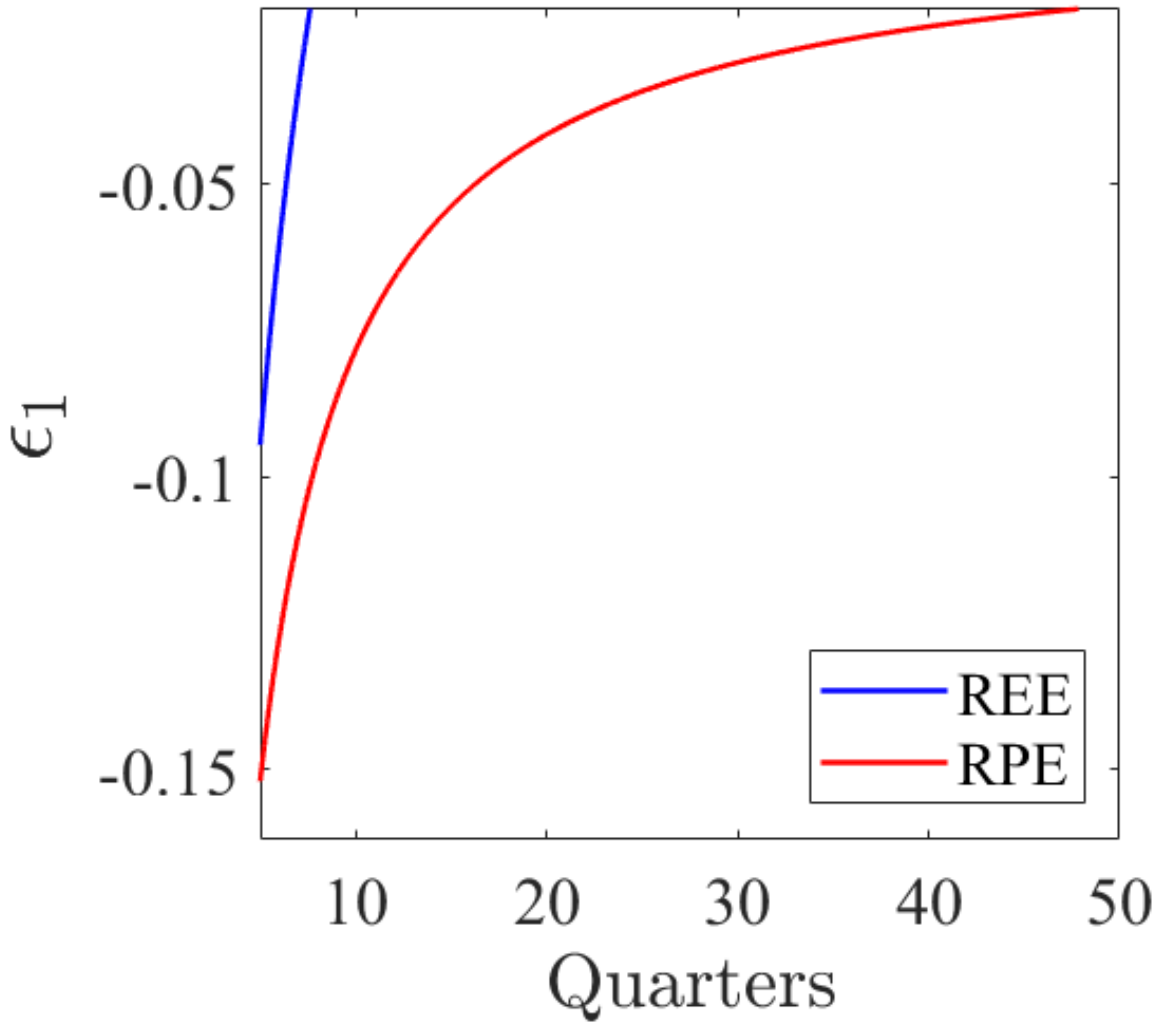
Is the RPE Reasonable?

2. Trying to learn the 2-state process generates deflationary spirals under incoherence.



Is the RPE Reasonable?

3. Expected duration of ZLB in a MSV is implausibly short



Is the RPE Reasonable?

4. RPE can be derived under more general assumptions.
   a. Incoherent non-linear model (e.g. Bianchi, Melosi, Rottner, 2021) admits RPE.
   b. Analytical RPE existence result with continuous AR shock process.
      - Two RPE exist if shock variance is sufficiently small.
      - Both RPE feature deflationary bias (average inflation below target).
      - RPE with higher average inflation is learnable (simulations).

RPE with Continuous Shocks