Optimal Monetary Policy with Heterogeneous Agents:
A Timeless Ramsey Approach

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Motivation

- Heterogeneity in households' exposure to business cycle fluctuations

- Monetary policy has distributional consequences
  - Mounting empirical evidence
    Doepke-Schneider (2006), Coibion et al. (2017), Clayton et al. (2018), Ampudia et al. (2018), ...
  - Important lesson from growing heterogeneous-agent New Keynesian ("HANK") literature

- Fed increasingly taking into account "distributional considerations"

  Our revised statement emphasizes that maximum employment is a broad-based and inclusive goal. This change reflects our appreciation for the benefits of a strong labor market, particularly for many in low- and moderate-income communities. — Jerome H. Powell, August 2020

Q: Implications of household heterogeneity for optimal monetary policy?

This Paper

- Timeless Ramsey approach to jointly characterize:
  1. Optimal long-run policy
  2. Time consistency and targeting rules
  3. Optimal stabilization policy

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  3. Under commitment, 0 inflation optimal long-run policy
  4. Standard inflation target now augmented by distributional considerations
  5. Time-consistent monetary policy requires a novel distributional target
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- Extend sequence-space approach to Ramsey problems and welfare analysis

  Boppart-Krusell-Mitman (2018), Auclert-Bardóczy-Rognlie-Straub (2021)

Model

Overview

- Minimal departure from standard New Keynesian ("RANK") model
  1. Incomplete markets + idiosyncratic risk *Huggett (1993)*
  2. Wage rigidity *Erceg et al. (2000), Auclert-Rognlie-Straub (2020)*

- Continuous time, $t \in [0, \infty)$

- No aggregate risk: focus on one-time, unanticipated shocks

- Types of shocks: Demand (discount rate) $\boldsymbol{\rho}_t$, supply (TFP) $\boldsymbol{A}_t$, and cost-push $\boldsymbol{\epsilon}_t$

Households

Preferences: Households' private lifetime utility is

$$V_0(\cdot) = \max \mathbb{E}_0 \int_0^\infty e^{-\int_0^t \boldsymbol{\rho}_s ds} \underbrace{U_t(c_t, n_t)}_{\text{Instantaneous Utility Flow}} dt$$

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Budget constraint: $\dot{a}_t = r_t a_t + z_t w_t n_t + \tau(z_t) - c_t$

- Households trade a bond a_t, borrowing constraint: $a_t \geq \underline{a}$
- Idiosyncratic labor productivity z_t: two-state Markov process
- Lump-sum rebate $\tau(z_t)$ (= 0 in equilibrium)

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Cross-sectional distribution: denote joint density $g_t(a, z)$

Labor Markets and Production

Off-the-shelf model of **nominal wage rigidity**: *Erceg et al. (2000), Auclert-Rognlie-Straub (2020)*

- Labor rationing: households work same hours, $n_t = N_t$
- New Keynesian wage Phillips curve:

$$\dot{\pi}_t^w = \rho_t \pi_t^w + \underbrace{\frac{\epsilon_t}{\delta}}_{\text{NKPC slope}} \iint n_t \Bigg(\underbrace{\frac{\epsilon_t - 1}{\epsilon_t} \overbrace{(1+\tau^L)}^{\text{Employment Subsidy}}}_{\text{Desired Markup}} \underbrace{w_t z u'(c_t) \; - \; v'(n_t)}_{\text{Individual Labor Wedge: } \tau_t(a,z)} \Bigg) g_t(a,z) \, da \, dz$$

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Production: representative firm produces consumption good $Y_t = A_t N_t$

- Perfect competition + flexible prices: $\frac{W_t}{P_t} = w_t = A_t$ (wages = MRT $\neq$ MRS)

Remaining Model Details

Government:

- Fiscal authority: pays for employment subsidy with lump-sum tax
- Policy instrument: path of interest rates $\{i_t\}_{t\geq 0}$

Market clearing: Goods: $$Y_t = C_t = \iint c_t(a,z) g_t(a,z)\, da\, dz$$

Bonds: $$0 = B_t = \iint a g_t(a,z)\, da\, dz$$

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Sources of suboptimality:

(1) Monopolistic competition

(2) Nominal rigidity

(3) Labor rationing

(4) Incomplete markets

Planning Problem

Primal approach: planner picks among implementable competitive equilibria

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The **Standard Primal Ramsey Problem** solves: $\max L(g_0)$, where

$$L = \int_0^\infty e^{-\int_0^t \rho_s ds} \Big\{ \iint \underbrace{\omega_t(a,z)}_{\text{welfare weights}} U_t(a,z)\, g_t(a,z)\, da\, dz \; +$$

Primal approach: planner picks among implementable competitive equilibria

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Planner faces **5 implementability conditions**: **micro block**

$$u'(c_t(a,z)) = \partial_a V_t(a,z) \qquad \mathbf{FOC}_t(a,z)$$

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$$\frac{d}{dt} g_t(a,z) = \Lambda_t^{\mathrm{KFE}} g_t(a,z) \qquad \mathbf{KFE}_t(a,z)$$

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macro : $$+ \mu_t \mathbf{RC}_t + \theta_t \mathbf{NKPC}_t \Big\} dt$$

Planner faces **5 implementability conditions**: **macro block**

$$0 = \iint c_t(a,z) g_t(a,z)\, da\, dz - A_t N_t \qquad \mathbf{RC}_t$$

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A **Ramsey plan** is a solution to this problem, i.e., time paths for:

- Allocations and prices: $\{c_t(a,z),\, V_t(a,z),\, g_t(a,z),\, \pi_t^w,\, N_t\}_{t\geq 0}$
- Policy: $\{i_t\}_{t\geq 0}$
- Multipliers: $\{\phi_t(a,z),\, \chi_t(a,z),\, \lambda_t(a,z),\, \mu_t,\, \theta_t\}_{t\geq 0}$

Policy Under Discretion

Discretion: control over policy in "*present*", taking "*future*" (and expectations) as given

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Proposition. **(Non-Linear Targeting Rule for Policy under Discretion)**

$$\underbrace{\iint \left(z u'(c_t(a,z)) - \frac{v'(N_t)}{A_t} \right) g_t(a,z)\, da\, dz}_{\text{Aggregate Labor Wedge}} = \Omega_t \underbrace{\iint a u'(c_t(a,z)) g_t(a,z)\, da\, dz}_{\text{Distributive Pecuniary Effect}}$$

- Optimal policy trades off **1.** aggregate stabilization (LHS) against **2.** redistribution (RHS)
- Novel force: interest rate policy has **distributive pecuniary effect**
- Aggregate labor wedge < 0 at an optimum: $\iint a u'(c_t) g_t \, da\, dz = \mathbb{C}ov_{g_t}(a, u'(c_t)) < 0$
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Mechanism / **intuition**:

- Planner wants to lower real interest rates for redistribution
- Nominal rigidities: lower $i_t \implies$ lower $r_t \implies$ overheated economy with higher inflation

With isoelastic preferences, $u(c) = \frac{1}{1-\gamma} c^{1-\gamma}$ and $v(n) = \frac{1}{1+\eta} n^{1+\eta}$,

$$Y_t = \tilde{Y}_t \times \underbrace{\left(\frac{\epsilon_t}{\epsilon_t - 1} \frac{1}{1 + \tau^L} \right)^{\frac{1}{\gamma+\eta}}}_{\text{Markup Distortion}} \times \underbrace{\left(1 - \Omega_t \frac{\iint a u'(c_t(a,z)) g_t(a,z) \, da \, dz}{\iint z u'(c_t(a,z)) g_t(a,z) \, da \, dz} \right)^{\frac{1}{\gamma+\eta}}}_{\text{Redistribution}}$$

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RANK:	≥ 1	$= 1$
HANK:	≥ 1	> 1

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Proposition. In steady state, policy under discretion leads to **inflationary bias**:

$$\pi^w_{ss} = \frac{\epsilon}{\delta} A_{ss} N_{ss} \Bigg[\underbrace{\left(1 - \frac{\epsilon - 1}{\epsilon} (1 + \tau^L) \right) \Lambda_{ss}}_{\text{Markup Distortion: } \geq 0} \underbrace{- \; \Omega_{ss} \, \mathbb{C}ov_{g_{ss}(a,z)} \Big(a, u'(c_{ss}(a,z)) \Big)}_{\text{Redistribution: } > 0} \Bigg]$$

- Redistribution motive exacerbates inflationary bias: $4\times$ markup distortion term
- **HANK**: Gains from commitment even with appropriate employment subsidy

Timeless Ramsey Approach

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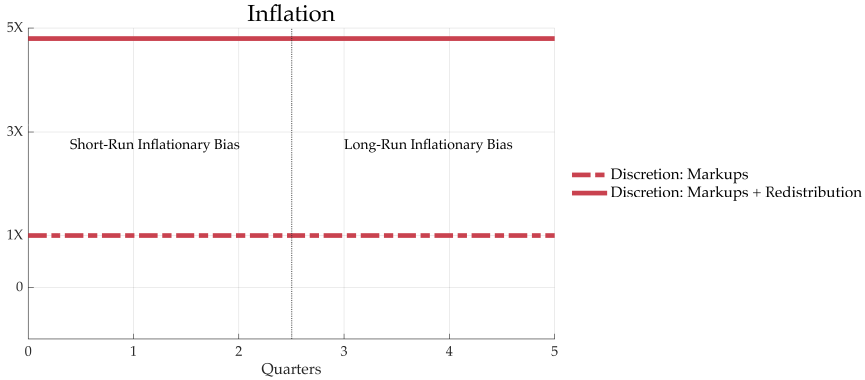

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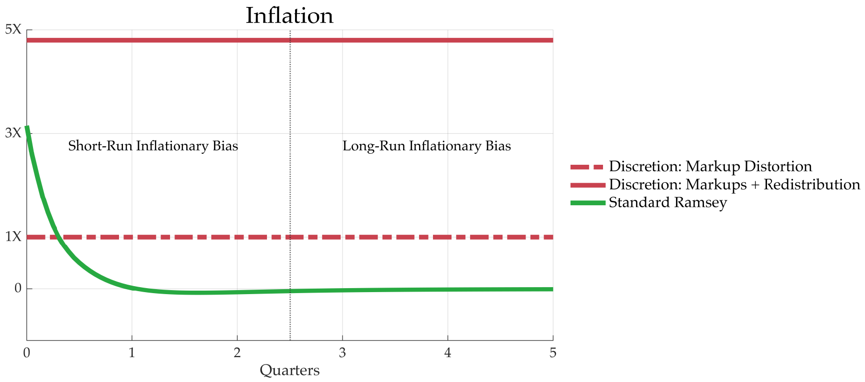
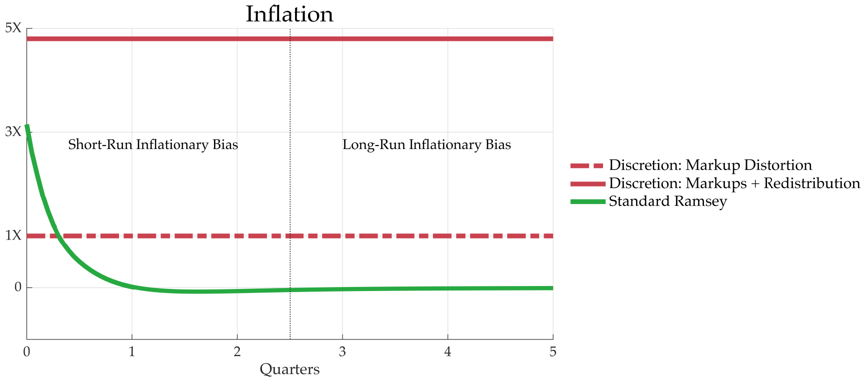


Step 1: optimal long-run inflation policy

- Policy under commitment converges to 0 inflation
- Standard Ramsey problem resolves inflationary bias in **long run**

Timeless Ramsey Approach



Step 2: time consistency and targets

- We still have inflationary bias in the short run!
- Two forward-looking constraints $\Longrightarrow$ planner wants to make promises $\Longrightarrow$ at time 0, no past promises $\Longrightarrow$ **time inconsistency**

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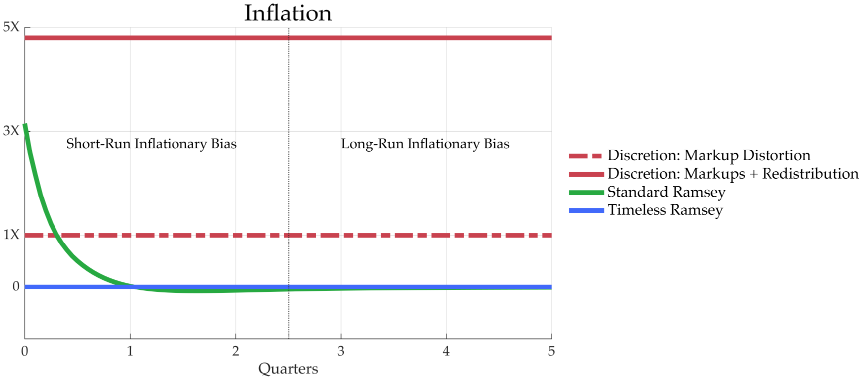

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- **Timeless Ramsey problem**: targeting rule to make policy time consistent

Step 2: Timeless Ramsey Problem

Definition. (**Timeless Penalties**) We define *timeless penalties* as

$$\mathcal{T}(\boldsymbol{\phi}, \boldsymbol{\theta}) = \underbrace{\iint \boldsymbol{\phi}(a,z) V_0(a,z)\, da\, dz}_{\text{Distributional Target}} \quad - \quad \underbrace{\boldsymbol{\theta} \pi_0^w}_{\text{Inflation Target}}$$

$\Longrightarrow$ Generalizes Marcet-Marimon (2019) to continuous-time heterogeneous-agent economies

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Proposition. Policy under the Timeless Primal Ramsey Problem is time consistent

No inflationary bias in short run or long run

Inflation Target

Proposition. (**Inflation Target**) Timeless penalty takes form of an inflation target:

$$\underbrace{-\theta_{\mathrm{ss}} \pi_0^w}_{\text{Linear inflation target (Walsh, 1995)}}, \quad \text{where} \qquad \theta_{\mathrm{ss}} = -\frac{\delta}{\epsilon} \frac{\boldsymbol{\Omega}_{\mathrm{ss}}^1 - Y_{\mathrm{ss}}^{\gamma+\eta}}{\frac{\epsilon-1}{\epsilon}(1+\tau^L)(1-\gamma)\boldsymbol{\Omega}_{\mathrm{ss}}^2 - (1+\eta) Y_{\mathrm{ss}}^{\gamma+\eta}}$$

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HANK:

- Distributional considerations impact inflation target
- Even with employment subsidy, $\theta_{ss} \neq 0$

Distributional Target

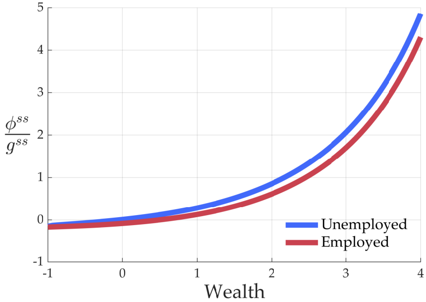
Proposition. (**Distributional Target**) In HANK, new distributional target required:

$$\iint \phi_{\text{ss}}(a,z) V_0(a,z)\, da\, dz$$

- Distributional target solves "**promise-keeping Kolmogorov forward equation**":

$$0 = \Lambda_{\text{ss}}^{\text{KFE}} \phi_{\text{ss}}(a,z) + \partial_a \chi_{\text{ss}}(a,z)$$

- Planner's promise not to surprise-redistribute is not time consistent
- Like inflation target but for redistribution: $\phi_{\text{ss}}(a,z) < 0$ for the poor



Step 3: Optimal Stabilization Policy

Proposition. **(Non-Linear Targeting Rule for Stabilization Policy)**

$$Y_t = \tilde{Y}_t \times \left\{ \frac{\frac{\epsilon_t}{\epsilon_t - 1} \frac{1}{1+\tau^L} \boldsymbol{\Omega}_t^1 + \theta_t (1-\gamma) \frac{\epsilon_t}{\delta} \boldsymbol{\Omega}_t^2}{1 + \theta_t (1+\eta) \frac{\epsilon_t}{\delta}} \right\}^{\frac{1}{\gamma+\eta}}$$

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RANK: $\boldsymbol{\Omega}_t^1, \boldsymbol{\Omega}_t^2 \to 1 \quad \Longrightarrow \quad$ Divine Coincidence if $\frac{\epsilon_t - 1}{\epsilon_t}(1 + \tau^L) = 1$

- **Demand** / **TFP** shock: $\pi_t^w = 0 \quad \Longrightarrow \quad \theta_t = 0 \quad \Longrightarrow \quad Y_t = \tilde{Y}_t$
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HANK: Divine coincidence generically fails

- Trade-off between inflation / output (aggregate efficiency) and distributional considerations
- Accounting for "**distributional considerations**" comes at cost of **aggregate efficiency**

Sequence-Space Approach to Ramsey Problems

- Extend sequence-space apparatus to optimal policy and welfare analysis
  Build on Auclert-Bardóczy-Rognlie-Straub (2021)
- Notation: Path of policy $\boldsymbol{i} = \{i_t\}_{t\geq 0}$, shocks $\boldsymbol{Z} = \{A_t, \rho_t, \epsilon_t\}_{t\geq 0}$, macro aggregates $\boldsymbol{X} = \{X_t\}_{t\geq 0}$, and aggregate multipliers $\boldsymbol{M} = \{\theta_t, \mu_t\}_{t\geq 0}$

Proposition. (**Sequence-Space Representation of Ramsey Plans**) Given g_0, initial promises $\boldsymbol{\phi}$ and $\boldsymbol{\theta}$, and path of shocks $\boldsymbol{Z}$, a Ramsey plan $\boldsymbol{R} = (\boldsymbol{X}, \boldsymbol{M}, \boldsymbol{i})$ solves

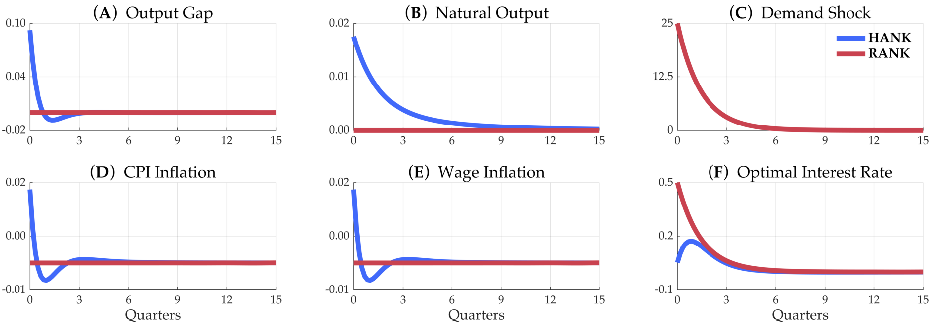
$$\mathcal{R}(\boldsymbol{X}, \boldsymbol{M}, \boldsymbol{i}, \boldsymbol{Z}) = 0 \quad \Longrightarrow \quad \boldsymbol{R} = \boldsymbol{R}(\boldsymbol{Z}; g_0, \boldsymbol{\phi}, \boldsymbol{\theta})$$

Proposition. (**Sequence-Space Perturbations**)

$$d\boldsymbol{R} = -\mathcal{R}_{\boldsymbol{R}}^{-1}\, \mathcal{R}_{\boldsymbol{Z}}\, d\boldsymbol{Z}$$

- $\mathcal{R}_{\boldsymbol{R}}$ and $\mathcal{R}_{\boldsymbol{Z}}$ are Jacobians of the Ramsey map $\Longrightarrow$ extend ABRS fake-news algorithm
- Timeless approach absolutely critical for validity of first-order approximation

Demand Shock



Calibration: $\rho = 0.02 \quad \gamma = \eta = 2 \quad z \in \{0.8, 1.2\} \quad \epsilon = 10 \quad \delta = 100$

Conclusion

- Paper revisits New Keynesian consensus on optimal monetary policy in HANK

- Discretion: novel redistribution motive exacerbates inflationary bias

- Commitment: **Timeless Ramsey approach** to jointly study
  1. Optimal long-run policy
  2. Time consistency and targeting rules $\rightarrow$ distributional target needed
  3. Optimal stabilization policy

- Extend sequence-space apparatus to Ramsey problems and welfare analysis

- FOC for $g_t(a,z)$ defines **social lifetime value** $\lambda_t(a,z)$ with Bellman:

$$\rho\lambda_t(a,z) = U_t(a,z) + \mathbb{E}_t\left[\frac{d\lambda_t(a,z)}{\lambda_t(a,z)}\right] + \underbrace{\mu_t\Big(c_t(a,z) - A_t z n_t\Big)}_{\text{Individual Contribution to Aggregate Excess Demand}} \quad + \quad \underbrace{\theta_t \frac{\epsilon_t}{\delta}\tau_t(a,z)}_{\text{Individual Contribution to Inflation}}$$

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- FOC for interest rate i_t:

$$0 = \iint \Big(\underbrace{a\,\partial_a\lambda_t(a,z)\,g_t(a,z)}_{\substack{\text{Distributive Pecuniary Effect} \\ \text{+ Spending on Externalities}}} \; + \; a\,\partial_a V_t(a,z)\underbrace{\phi_t(a,z)}_{\text{“Distributional Penalty”}} \Big)\, da\, dz$$

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- Evolution of inflation penalty:

$$\underbrace{\dot{\theta}_t}_{\text{“Inflation Penalty”}} = \delta\pi_t^w + \underbrace{\iint \Big(a\,\partial_a\lambda_t(a,z)\,g_t(a,z) + a\,\partial_a V_t(a,z)\,\phi_t(a,z)\Big)\, da\, dz}_{=0}$$

To characterize new inflation target $\implies$ summary statistics for role of heterogeneity

Definition. (Distributional Wedges)

$$\Omega_t^1 = \iint \left(\frac{zu'(c_t)}{u'(Y_t)} + \frac{zu'(c_t)}{u'(Y_t)} \frac{\phi_t}{g_t} + \frac{zu''(c_t)}{u'(Y_t)} \frac{\chi_t}{g_t} \right) g_t \, da \, dz$$

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Aggregate efficiency planner / mandate: $\Omega_{1,t}, \Omega_{2,t} \to 1$ *Dávila-Schaab (2021)*

Optimal Long-Run Inflation Policy

RANK: $$\dot{\vartheta}_t^{\text{RA}} = \delta \pi_t^{w,\text{RA}}$$

HANK: $$\dot{\vartheta}_t = \delta \pi_t^w + \iint \Big(a\phi_t(a,z)\partial_a V_t(a,z) + a g_t(a,z)\partial_a \lambda_t(a,z) \Big)\, da\, dz$$

Proposition. First-order condition for optimal monetary policy in **HANK**

$$0 = \iint \Big(a\phi_t(a,z)\partial_a V_t(a,z) + a g_t(a,z)\partial_a \lambda_t(a,z) \Big)\, da\, dz$$

- Baseline **HANK** agrees with **RANK** on 0 optimal long-run inflation
- "Necessary condition" for HA to imply non-zero optimal inflation:
  Distributional consequences of inflation must be partly orthogonal to nominal interest rate
- Baseline model does not have alternative motives for long-run inflation
  Khan-King-Wolman (2003), Schmitt-Grohé-Uribe (2010)

$\Longrightarrow$ our approach applies to settings with distributional consequences of long-run inflation

Proposition 1. **(Ramsey Plan)**

a) First-order necessary conditions:

$$\boldsymbol{g}: \quad \boldsymbol{\rho}_t \lambda_t(a,z) = u(c_t) - v(N_t) - \frac{\delta}{2}(\pi_t^2) + \mathcal{A}_t \lambda_t - \mu_t c_t + \vartheta_t \frac{\boldsymbol{\epsilon}_t}{\delta} \frac{\boldsymbol{\epsilon}_t - 1}{\boldsymbol{\epsilon}_t}(1+\tau^L)\boldsymbol{A}_t N_t z u'(c_t)$$

$$\boldsymbol{V}: \quad 0 = \mathcal{A}_t^* \phi_t(a,z) + \partial_a \chi_t(a,z)$$

$$\boldsymbol{c}: \quad \chi_t(a,z) = -\frac{g_t}{u''(c_t)} \left[\begin{array}{c} u'(c_t) - \mu_t - \partial_a \lambda_t(a,z) \\ +\vartheta_t \frac{\boldsymbol{\epsilon}_t}{\delta} \frac{\boldsymbol{\epsilon}_t - 1}{\boldsymbol{\epsilon}_t}(1+\tau^L)\boldsymbol{A}_t N_t z u''(c_t) \end{array} \right]$$

$$\boldsymbol{N}: \quad 0 = \mu_t - \frac{1}{\boldsymbol{A}_t} v'(N_t) + \vartheta_t \frac{\boldsymbol{\epsilon}_t}{\delta}\left[\frac{\boldsymbol{\epsilon}_t - 1}{\boldsymbol{\epsilon}_t}(1+\tau^L)\boldsymbol{A}_t \Lambda_t - v'(N_t) - v''(N_t)N_t\right] + \iint \Big(z\phi_t(a,z)\partial_a V_t(a,z) + z g_t(a,z)\partial_a \lambda_t(a,z)\Big)\, da\, dz$$

$$\boldsymbol{\pi}^w: \quad \dot{\vartheta}_t = \delta \pi_t^w$$

$$\boldsymbol{i}: \quad 0 = \iint \Big(a\phi_t(a,z)\partial_a V_t(a,z) + a g_t(a,z)\partial_a \lambda_t(a,z)\Big)\, da\, dz$$

b) Initial conditions: **(1)** $\vartheta_0 = 0$ **(2)** $\phi_0(a,z) = 0$

Implementability Conditions

- **Primal approach**: planner picks among implementable competitive equilibria

  Paper also characterizes dual approach

- Find *minimal* set of implementability conditions, associate Lagrange multiplier with each

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Micro block:

$$\rho_t V_t(a,z) = U_t(a,z) + \mathbb{E}_t\left[\frac{dV_t(a,z)}{dt}\right] \qquad \phi_t(a,z)\mathbf{HJB}_t(a,z)$$

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Macro block:

$$0 = A_t N_t - \iint c_t(a,z) g_t(a,z)\, da\, dz \qquad \mathbf{RC}_t$$

$$\dot{\pi}_t^w = \rho_t \pi_t^w + \frac{\epsilon_t}{\delta}\left[\frac{\epsilon_t - 1}{\epsilon_t}(1+\tau^L) w_t \iint z u'(c_t) g_t(a,z)\, da\, dz - v'(N_t)\right] N_t \qquad \mathbf{NKPC}_t$$

Model Benchmarks and Calibration

Today: main comparison benchmark **RANK limit**

- Limit of no earnings risk: $z_t \to^P \bar{z} = 1$
- Initialize economy at $g_0(a, z) =$ Dirac mass point at $(a, z) = (0, \bar{z})$
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Skip today: planners in **HANK** and **RANK** agree on 0 optimal long-run inflation